# Competitive sequential search equilibrium*

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**Abstract**

We present a tractable model of a competitive equilibrium where buyers engage in costly sequential search for a satisfactory match. The socially optimal consumer search requires buyers not to be too picky, which implies that the tension between increasing the likelihood of a match at the expense of lowering the quality of the match is resolved in favour of the former. As a result, a higher search cost increases welfare. In a free entry equilibrium, this result is translated into an insufficient level of entry. For a range of search costs, seller entry attains higher welfare with random search than with competitive search.

**Keywords**: Competitive search, Search costs, Sequential search, Entry

**JEL Classification Number**: D4, J6, L1, L8, R3

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# 1 Introduction

In many consumer search markets firms are capacity constrained. For example, shoes and clothing retail outlets can only stock a limited number of units and it is not uncommon that consumers are rationed at these shops and need to search at competing outlets. Similarly, hotels have a limited number of rooms, airlines a limited number of seats, restaurants a limited number of tables and economics departments a few assistant professor positions.

The study of pricing of capacity constrained firms in consumer search markets has received attention at least since Peters (1984). Peters considers a homogeneous product market where capacity constrained firms compete in prices and consumers can only search once. Because of the assumption of standardized products, Peters deviates from the standard search paradigm by assuming that firm prices are observable by consumers before visiting firms. Otherwise, without ex-ante competition, a Diamond's (1971)-like result would naturally arise, possibly leading to a market collapse. Peters (1984) demonstrates that a price-dispersed equilibrium exists where buyers adopt mixed search strategies and firms randomize their prices. Peters (1991) extends the analysis by allowing agents who match to exit the market. His focus is the derivation of the matching technology.

An important literature that follows Peters (1984, 1991) has revolved around the question whether entry is efficient and has kept the focus on markets for homogeneous products and ex-ante competition. In his seminal contribution, Moen (1997) demonstrates that seller entry is efficient in competitive search equilibrium.

The focus on homogeneous products is somewhat restrictive. Most products are differentiated. Moreover, products differ in characteristics that can hardly be observed from home. At the same time, there are characteristics that can hardly be advertised. Unlike in the standard competitive search literature, in this type of markets the purpose of search is not just finding whether a product is available, but also whether the product suits the needs of the consumer. Think of the job market for economists. A important part of the process is to find out whether a candidate is god match for a department.

In this paper, we propose to study the role of capacity constraints in consumer search markets for differentiated products. To the best of our knowledge, dealing with product differentiation is new in the competitive search equilibrium literature. Modelling product differentiation has the advantage that we do not need to assume ex-ante competition in order to get rid of the Diamond's paradoxical result. The reason is that consumers do not search just for good prices but for satisfactory products, which implies that in a putative equilibrium where all sellers sell at monopoly prices, consumers still do have an incentive to continue to search in order to find better products. The framework then naturally lends itself to compare ex-ante and ex-post competition. We are interested in whether inefficient entry occurs in this setting, which implies that the price cannot induce the right amount of sequential search by buyers.

Our model adapts the Wolinsky's (1986) framework to settings where firms are capacity constrained.[1] Specifically, we study the equilibrium of a large market with many sellers and many buyers. Sellers sell differentiated products and consumers search to find products that are satisfactory. Sellers are capacity constrained in the sense that they can sell a maximum of one unit per search period. Consumers randomly visit sellers and choose to continue searching in case they are rationed or alternatively they do not like the product. Consumers who match leave the market and, to maintain market stationarity, are replaced by new consumers. We show that there exists a unique symmetric pure strategy Nash equilibrium in this market.

As in standard consumer search models, an increase in search costs results in higher prices. More interesting is the behavior of the equilibrium price with respect to the buyer-seller ratio. An increase in the number of buyers per firm has two effects. On the one hand, demand becomes less elastic because the chance that a firm sells its unit increases. On the other hand, consumers, because the chance they are offered the product of a firm goes down, become less picky and are ready to accept worse products. Both these effects tend to increase prices.

Surprisingly, welfare is increasing in search costs. Welfare depends on the quality of the match

[1] Wolinsky's paper has led to a stream of contributions in the consumer search literature and could well be referred to as the workhorse model of consumer search for differentiated products. Recent contributions study incentives to invest in quality (Wolinsky, 2005), product-design differentiation (Bar-Isaac et al., 2011), the emergence and effects of market prominence (Armstrong et al., 2009; Armstrong and Zhou, 2011; Haan and Moraga-González, 2011), multi-product search (Zhou, 2014) and vertical relations (Janssen and Shelegia, forthcoming).

and the probability of matching. What happens is that as search costs increase, the probability consumers match with products goes up; however, with higher search costs consumers are led to accept worse matches. We show that the first effect dominates. We also study a long-run version of our model where we allow for free entry. The equilibrium number of sellers increases in search cost and decreases in entry cost.

We finally move to study whether seller entry is efficient in our setting. Interestingly, we prove that the free entry number of firms is excessive from the perspective of social welfare maximization, provided search costs are sufficiently large. Numerical simulations of the model suggest that entry is insufficient when the search cost is sufficiently small. With ex-post competition, then, entry is generically inefficient. With ex-ante competition entry can never be insufficient and is only optimal in the limit when the search cost converge to the highest admissible search cost level.

The rest of the paper is organized as follows. We introduce a two-period version of our model in section 2 with the purpose of illustrating the central tradeoff between match quality and match probability that we have discussed above. In Section 3 we present the infinite horizon version of our model, derive both the random and directed search equilibria and the main comparative statics results. The consequences of free entry and the generic inefficiency of (random and directed) search equilibrium are discussed in Section 4. We conclude in section 5. An appendix contains some auxiliary derivations.

## 2 Model

We consider a consumer search market with *horizontally differentiated products*, as in Wolinsky (1986, 1988).[2] There is a measure $B$ of buyers, and a measure $S$ of sellers. Let $x$ be the buyer-seller ratio, i.e. $x \equiv B/S$. Each seller has a selling capacity equal to one; this means that each seller can only sell a maximum of one unit of its product in a given (search) period. For simplicity, we normalize unit production costs to zero. Firms compete in prices. We study both cases, the case in which firm prices are not observed before search (random search) and the case in which they are (directed search).[3]

Each buyer has unit demand. The exact value a consumer $\ell$ places on the product of a firm $i$, denoted $\varepsilon_{i\ell}$, depends on how well the product matches the tastes of the consumer. Such a match value can only be learnt upon inspection of the product. We assume that match values are identically and independently distributed across buyers and sellers. Let $F$ be the distribution of match values, with density $f$ and support $[0, \bar{\varepsilon}]$. Following the literature, we assume the density $f$ to be log-concave. From now on, we drop the sub-index of $\varepsilon_{i\ell}$.

Buyers have to visit stores in order to inspect products and learn how much they value them. We refer to this activity as search. Consumers search sequentially. Every time a buyer searches, she has to pay a search cost $c > 0$. Because sellers' capacity is limited, once the buyer arrives at the store of a seller, she may or may not get an opportunity to inspect the product. The first event occurs when the seller directly picks the consumer in question and offers the product to her; or when the seller picks the consumer after having offered the product to some other buyers who in turn decide to search on. In case the consumer is offered the product, she has to decide whether to buy it or, alternatively, search on. The second event occurs when the seller chooses to offer the product to someone else in its queue of buyers and such a buyer chooses to acquire the product.

[2] As will become clear later, the fact that sellers sell differentiated products is central to our model and distinguishes our work from the seminal paper of Moen (1997). When products are horizontally differentiated buyers differ in the way the rank the various products. When products are homogeneous, even though buyers may differ from one another in their valuations, each buyer allocates the same value to all the products.

[3] A nice feature of our model with horizontally differentiated products and capacity constraints is that we can compare these two modes of competition in a common framework. To the best of our knowledge, no other models lend themselves to this comparison in an interesting and tractable way. For example, allowing for ex-ante competition in the model of Wolinsky's leads to marginal cost pricing while modelling random search in the homogeneous products case leads to the Diamond paradox.

We assume that sellers pick buyers randomly.

Because firms products are ex-ante symmetric, consumers will search randomly in situations where prices are not observable before search. We refer to this as the random search model. While searching randomly, we assume that consumers hold correct conjectures about the equilibrium price. When consumers do observe prices before search, search will be directed. We refer to this case as the directed search model.

## 3 A two-period example

To illustrate an essential ingredient of our theory, namely that social welfare is non-monotonic in search costs, we first study a two-period version of our model. Moreover, we assume that buyers and sellers that match in the first period leave the market. Unmatched buyers, i.e. those who fail to get an opportunity to buy plus those who endogenously choose not to buy, and unmatched sellers, i.e. those who fail to sell in the first period, go to the second and final period. In this simplified economy, total demand and supply are fixed.

As we will see later, whether search is random or directed will not matter for the welfare analysis because prices drop from the welfare formula. Nevertheless, to fix ideas and notation, let us consider the case in which prices are not observable so search is random.

In standard sequential search environments, it is well known that the optimal search strategy consists of a stopping rule (see e.g. McCall, 1970). Such a stopping rule prescribes the consumer to check in every search period $t$ whether the product at hand provides the buyer with a match value above a reservation value, in which case the buyer should stop search and buy it; otherwise the buyer should search on. Let us denote the reservation value in period $t$ as $\hat{\varepsilon}_t$, $t = 1, 2$.

Let $x_t$ the buyer-seller ratio in period $t$. Obviously $x_1 = x$. The buyer-seller ratio in period 2, $x_2$, depends on how many buyers and sellers remain unmatched in period 2; this will be computed later. Let $\bar{x}_t$ denote the *effective queue* of buyers in period $t$, i.e., the expected number of buyers who will show up at a typical firm $i$ *and* would stop searching and buy there if they were given the option to inspect the product of firm $i$. Interestingly, the effective queue in a given period $\bar{x}_t$ crucially depends on the search cost because when buyers walk more easily away from a given seller, then it is more likely that a given buyer in the queue of a seller is offered the product. Obviously, the effective queue of buyers in period $t$ also depends on period $t$ buyer-seller ratio.

Let us denote the probability with which a buyer in the queue of a typical seller gets an opportunity to inspect and buy the product by $\eta(\bar{x}_t)$. In order to compute $\eta(\bar{x}_t)$, we first note that, because buyers search the market randomly, the number of buyers $n$ that visit a randomly selected seller follows (in large markets) a Poisson distribution, with the Poisson parameter equal to the effective buyer-seller ratio. Therefore, the probability that $k$ consumers show up at a given seller in period $t$ is given by

$$\Pr(n_t = k) = \frac{\bar{x}_t^k e^{-\bar{x}_t}}{k!},$$

where Pr indicates "probability." Using this, in the Appendix we compute the probability with which a buyer in the queue of a typical seller gets an opportunity to inspect and buy the product:

$$\eta(\bar{x}_t) = \frac{1 - e^{-\bar{x}_t}}{\bar{x}_t}. \tag{1}$$

This functional form is similar to that in related models of retail search markets (see e.g. Butters (1977) and Peters (1984, 1991, 2001)) except that we have here the effective queue $\bar{x}_t$ rather than just the queue $x_t$. The effective queues in periods 1 and 2 are given by:

$$\bar{x}_1 \equiv x_1(1 - F(\hat{\varepsilon}_1)) = x(1 - F(\hat{\varepsilon}_1)) \tag{2}$$

$$\bar{x}_2 \equiv x_2(1 - F(\hat{\varepsilon}_2)) = \frac{1 - \eta(\bar{x}_1)(1 - F(\hat{\varepsilon}_1))}{1 - \bar{x}_1\eta(\bar{x}_1)}(1 - F(\hat{\varepsilon}_2)). \tag{3}$$

As mentioned before, the effective queues capture the fact that products are differentiated and consumers sometimes discard the product at hand and search again. Because of this, the number of consumers who will effectively demand the product of a firm in period 1 is $B[1 - F(\hat{\varepsilon}_1)]$. In period 2, given the exit of matched agents, there will be $B[1 - \eta(\bar{x}_1)(1 - F(\hat{\varepsilon}_1))]$ buyers still in the market, and a proportion $1 - F(\hat{\varepsilon}_2)$ of them will find the product acceptable; meanwhile, there will be only $S(1 - \bar{x}_1\eta(\bar{x}_1))$ sellers.

### Welfare

We now derive welfare in our two-periods economy. Fix the strategies of the firms, i.e. prices $p_1$ and $p_2$ for periods 1 and 2, and let us start with the buyers' search problem. Upon paying search cost $c$, a buyer expects to obtain gross utility $E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_1)$ conditional on successfully matching in period 1, which occurs with probability $\eta(\bar{x}_1)(1-F(\hat{\varepsilon}_1))$. Otherwise, she will proceed to the next period, in which case she will have to pay the search cost again in exchange for possibly getting another opportunity to inspect a product and get matched. Hence, the expected utility to a buyer that searches the market for a satisfactory match is

$$\begin{aligned} V &= -c+\eta(\bar{x}_1)(1-F(\hat{\varepsilon}_1))[E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_1)-p_1] \\ &\quad +\beta\left[1-\eta(\bar{x}_1)(1-F(\hat{\varepsilon}_1))\right]\left[-c+\eta(\bar{x}_2)(1-F(\hat{\varepsilon}_2))[E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_2)-p_2]\right], \end{aligned}$$

where $\beta$ is a discount factor.

The sellers' expected profits are

$$\Pi = \bar{x}_1\eta(\bar{x}_1)p_1 + \beta(1-\bar{x}_1\eta(\bar{x}_1))\bar{x}_2\eta(\bar{x}_2)p_2,$$

since a seller attracts $x_t$ buyers on average and sells his unit successfully with probability $x_t\eta(x_t)(1-F(\hat{\varepsilon}_t))$. Clearly, the more buyers a seller attracts, the more likely he can sell successfully, hence the probability $x_t\eta(x_t)$ is increasing in the effective queue $x_t$.

We now calculate welfare. Using equations (2) and (3) and after rearranging, *ex ante* welfare is given by

$$\begin{aligned} W &= BV + S\Pi \\ &= S\{-cx[1+\beta(1-\eta(\bar{x}_1)(1-F(\hat{\varepsilon}_1))] \\ &\quad +\bar{x}_1\eta(\bar{x}_1)E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_1)+\beta(1-\bar{x}_1\eta(\bar{x}_1))\bar{x}_2\eta(\bar{x}_2)E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_2)\}. \end{aligned} \tag{4}$$

Note that aggregate welfare does not depend on prices. This means that the welfare expression is the same no matter whether search is random or search is (price-)directed.

Observe that

$$\frac{dW}{d\hat{\varepsilon}_2} = [S\beta(1-\bar{x}_1\eta(\bar{x}_1))]\frac{\partial[\bar{x}_2\eta(\bar{x}_2)E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_2)]}{\partial\hat{\varepsilon}_2}.$$

where

$$\frac{\partial[\bar{x}_2\eta(\bar{x}_2)E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_2)]}{\partial\hat{\varepsilon}_2} = \bar{x}_2\eta(\bar{x}_2)\frac{\partial E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_2)}{\partial\hat{\varepsilon}_2} + \frac{\partial\bar{x}_2\eta(\bar{x}_2)}{\partial\bar{x}_2}\frac{\partial\bar{x}_2}{\partial\hat{\varepsilon}_2}E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_2). \tag{5}$$

In this expression, the first term is positive since the higher the reservation value, the higher the expected consumption value. This improves the quality of the match between consumers and firms and hence welfare. On the other hand, the second term is negative since the probability with which a firm and a consumer match and trade $\bar{x}_2\eta(\bar{x}_2)$ is increasing in the effective queue $\bar{x}_2$, which decreases with $\hat{\varepsilon}_2$. The higher the reservation value, the less likely is that a trade occurs. This deteriorates the match probability and welfare. Hence, unless one effect dominates the other, when choosing the reservation value $\hat{\varepsilon}_2$ the planner faces a tradeoff between a higher match value and a lower likelihood of successful trade.

Observe also that, for any $\hat{\varepsilon}_1 \in [\underline{\varepsilon}, \bar{\varepsilon}]$, because $\bar{x}_1$ does not depend on $\hat{\varepsilon}_2$, we have

$$\left.\frac{dW}{d\hat{\varepsilon}_2}\right|_{\hat{\varepsilon}_2=\bar{\varepsilon}} = [S\beta(1-\bar{x}_1\eta(\bar{x}_1))] \left.\frac{\partial \bar{x}_2\eta(\bar{x}_2)}{\partial \bar{x}_2}\frac{\partial \bar{x}_2}{\partial \hat{\varepsilon}_2}E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_2)\right|_{\hat{\varepsilon}_2=\bar{\varepsilon}} < 0,$$

which implies that welfare decreases in $\hat{\varepsilon}_2$ in a neighborhood of $\hat{\varepsilon}_2 = \bar{\varepsilon}$. Therefore, unlike in standard models of consumer search, the socially optimal reservation value, denoted by $\hat{\varepsilon}_t^*$, has to be such that $\hat{\varepsilon}_2^* < \bar{\varepsilon}$. Socially optimal consumer search requires buyers to be not too picky for otherwise the match probability is compromised.

For the choice of $\hat{\varepsilon}_1$,

$$\begin{aligned}\frac{\partial W}{\partial \hat{\varepsilon}_1}S^{-1} &= -cx\beta\left[-\frac{\partial \eta(\bar{x}_1}{\partial \bar{x}_1}\frac{\partial \bar{x}_1}{\partial \hat{\varepsilon}_1}(1-F(\hat{\varepsilon}_1)) + \eta(\bar{x}_1)f(\hat{\varepsilon}_1)\right] \\ &\quad +\bar{x}_1\eta(\bar{x}_1)\frac{\partial E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_1)}{\partial \hat{\varepsilon}_1} + \frac{\partial \bar{x}_1\eta(\bar{x}_1)}{\partial \bar{x}_1}\frac{\partial \bar{x}_1}{\partial \hat{\varepsilon}_1}\left[E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_1) - \beta\bar{x}_2\eta(\bar{x}_2)E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_2)\right] \\ &\quad +\beta(1-\bar{x}_1\eta(\bar{x}_1))\frac{\partial \bar{x}_2\eta(\bar{x}_2)}{\partial \bar{x}_2}\frac{\partial \bar{x}_2}{\partial \hat{\varepsilon}_1}E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_2). \end{aligned} \tag{6}$$

The first term is related to the change in the match quality, the second term the change in the match probability and the third term the change in the the expected queue next period. Observe that

$$\begin{aligned}\left.\frac{\partial W}{\partial \hat{\varepsilon}_1}S^{-1}\right|_{\hat{\varepsilon}_1=\bar{\varepsilon}} &= -cx\beta f(\bar{\varepsilon}) + \left.\frac{\partial \bar{x}_1\eta(\bar{x}_1)}{\partial \bar{x}_1}\frac{\partial \bar{x}_1}{\partial \hat{\varepsilon}_1}\right|_{\hat{\varepsilon}_1=\bar{\varepsilon}}\left[\bar{\varepsilon} - \beta\bar{x}_2\eta(\bar{x}_2)E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_2)|_{\hat{\varepsilon}_1=\bar{\varepsilon}}\right] \\ &\quad +\beta\left.\frac{\partial \bar{x}_2\eta(\bar{x}_2)}{\partial \bar{x}_2}\frac{\partial \bar{x}_2}{\partial \hat{\varepsilon}_1}E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_2)\right|_{\hat{\varepsilon}_1=\bar{\varepsilon}}. \end{aligned} \tag{7}$$

The first terms is negative and the second term, as before, is also negative. However, the sign of the third term is ambiguous, what makes it difficult to sign equation (7).

Consider the special case where the number of consumers is equal to the number of sellers, i.e., $x = \frac{B}{S} = 1$. This is special in the sense that the unmatched ratio of consumers to buyers is constant across period, which implies that $\frac{\partial x_2}{\partial \hat{\varepsilon}_1} = 0$. In this special case it is clear that welfare is decreasing in a neighborhood of $\hat{\varepsilon}_1 = \bar{\varepsilon}$. By continuity, we can then find values of $x$ close to 1 that make the third term as small as possible and the sign of (7) is negative.

**Theorem 1** *In the two-periods version of our model, for any discount factor $\beta \in [0, 1]$, there exists a set of parameters $(B, S, c)$ for which the socially optimal reservation values $\hat{\varepsilon}_1^*$ and $\hat{\varepsilon}_2^*$ are strictly lower than $\bar{\varepsilon}$.*

The basic message we wish to put forward here is that when firms are capacity constrained the social planner faces a trade-off when choosing reservation values: making consumers pickier results in an increase in consumer surplus conditional on a match, but the probability of a successful match goes down. Socially optimal search requires buyers not to be too picky.

# 4 Search market equilibrium

In this section, we extend our setup to the infinity horizon and construct a search market equilibrium by endogenizing consumer and firm strategies, i.e. the reservation value and price. We consider two search technologies for buyers: random search and competitive search.

Before preceding, we make one simplifying assumption. In the last section, we demonstrated the planner's tradeoff between match quality and trade volume, as is presented in (5). Compared to it, (7) includes an additional term, the third term, which reflects a change in the second period queue in response to the first period reservation value. This term could be non zero in non stationary environments with the changing number of unmatched buyers and sellers across the search periods, but does not deliver much economic insight. We therefore make the following assumption to maintain the environment stationary.

**Assumption 1** *In every period the number of active buyers and the number of active sellers are the same.*

In practice, Assumption 1 signifies that if $n$ matches occur in a period then we add to the population of buyers $n$ new buyers. For a similar setup see e.g., Benabou (1988, 1989, 1992), Fishman (1992) and Watanabe (2008). In addition, we should note that every seller can produce as many units as she likes, but can only sell one unit per period. This implies that if a seller is matched in period $t$ with a buyer, then the seller stays active in period $t+1$ with a new unit on display (see., Watanabe, 2010, 2013, for a similar setup).

## 4.1 Random search

We start with the random search technologies, where buyers can observe the individual price of a seller only after visiting it.

Note that, given our stationary environment, the population ratio $x = \frac{B}{S}$ and the reservation value $\hat{\varepsilon}$ are constant over the search periods. All buyers use the same $\hat{\varepsilon}$ and have the same probability $1 - F(\hat{\varepsilon})$ of accepting an offer with price $p$.

The reservation value of buyers is determined as follows:

$$\hat{\varepsilon} - p = -c + \eta(\bar{x})(1 - F(\hat{\varepsilon}))E[\varepsilon - p|\varepsilon \geq \hat{\varepsilon}] + [1 - \eta(\bar{x})(1 - F(\hat{\varepsilon}))]V \tag{8}$$

where $V$ is the expected value of search. This equation is interpreted as follows. Consider a buyer who has visited some firms and the best available option at hand gives her utility $\hat{\varepsilon} - p$. This is on the L.H.S. On the R.H.S., we have an expected utility of searching one more time. Upon paying search costs $c$, the buyer (randomly) visits a seller from whom she expects to receive an offer with probability $\eta(\cdot)$. Then, the buyer will accept the offer with probability $1 - F(\hat{\varepsilon})$ in which case the expected payoff is $E[\varepsilon - p|\varepsilon \geq \hat{\varepsilon}]$. If not matched with the product, she will proceed to the next period and the expected value of search is $V$. Since the R.H.S. should equal $V$, it holds that

$$V = \hat{\varepsilon} - p.$$

Note that assuming a negative value for the outside option implies that the value of search need not always be positive. See e.g., Anderson and Renault (1999).

In writing the above formula (8), we assume there is no recall, i.e. when the buyer does not match with the product, she does not retain the previous (best) offer that yields $\hat{\varepsilon} - p$. However, it is well know that optimal search is the same irrespective of whether we allow costless recall or not in large markets (operated by many firms), see e.g. Lipmann and McCall (1976). We should also note that, when there are capacity constraints it is not so clear any more that recall is costless and any previous offers can be guaranteed for sure – in fact, it is hard to imagine why the seller has to wait till the buyer's return even though she knows that this buyer may find something better elsewhere and may never come back. Instead, it seems more reasonable to expect that buyers have to re-queue in order to get previous offers. All in all, assuming that a buyer who walks away from a seller loses the opportunity to buy at that seller the observed match value, is similar to assuming that the buyer has to re-sample at that seller.

The above formula (8) can again be rewritten in the more familiar form:

$$\int_{\hat{\varepsilon}}^{\bar{\varepsilon}} (\varepsilon - \hat{\varepsilon}) f(\varepsilon) d\varepsilon = \frac{c}{\eta(\bar{x}(\hat{\varepsilon}))} \tag{9}$$

where the R.H.S. can be interpreted as an effective search cost.

**Sellers' price setting:** The expected profit to a firm $i$ that deviates from $p$ by charging $p_i$ is

$$\pi(p_i, p) = p_i \bar{x}(\hat{\varepsilon})\eta(\bar{x}(\hat{\varepsilon})) = p_i \left[1 - e^{-x(1-F(\hat{\varepsilon}-p+p_i))}\right], \tag{10}$$

where the offer probability $\eta(\cdot)$ is given by (1), and $\bar{x} = x(1-F(\cdot))$ is an effective queue of buyers.

The first order condition is

$$1 - e^{-x(1-F(\hat{\varepsilon}-p+p_i))} - p_i e^{-x(1-F(\hat{\varepsilon}-p+p_i))} x f(\hat{\varepsilon} - p + p_i) = 0.$$

After applying symmetry we have:

$$1 - e^{-x(1-F(\hat{\varepsilon}))} - x f(\hat{\varepsilon}) p e^{-x(1-F(\hat{\varepsilon}))} = 0.$$

Solving for $p$ gives

$$p = \frac{1 - e^{-x(1-F(\hat{\varepsilon}))}}{x f(\hat{\varepsilon}) e^{-x(1-F(\hat{\varepsilon}))}}. \tag{11}$$

**Equilibrium:** A stationary search equilibrium in our capacity constrained economy is described by $(p, \hat{\varepsilon})$ satisfying (9) and (11).

**Theorem 2** *For any $x = \frac{B}{S} \in (0, \infty)$ and $c \in (0, \eta(x)E(\varepsilon))$, there exists a unique solution $(p, \hat{\varepsilon}) > 0$ to (9) and (11). When the density of match values is non-decreasing, the equilibrium exists and is unique.*

**Proof.** Consider the determination of $\hat{\varepsilon} > 0$ in (9).

$$\Omega(\hat{\varepsilon}) \equiv \eta(\bar{x}(\hat{\varepsilon})) \int_{\hat{\varepsilon}}^{\bar{\varepsilon}} (\varepsilon - \hat{\varepsilon}) f(\varepsilon) d\varepsilon = c.$$

Observe that

$$\frac{d\Omega(\hat{\varepsilon})}{d\hat{\varepsilon}} = \frac{1 - e^{-\bar{x}} - \bar{x}e^{-\bar{x}}}{\bar{x}^2} x f(\hat{\varepsilon}) \int_{\hat{\varepsilon}}^{\bar{\varepsilon}} (\varepsilon - \hat{\varepsilon}) f(\varepsilon) d\varepsilon - \eta(\bar{x}) \int_{\hat{\varepsilon}}^{\bar{\varepsilon}} f(\varepsilon) d\varepsilon.$$

Arranging this derivative, we have

$$\begin{aligned} \frac{d\Omega(\hat{\varepsilon})}{d\hat{\varepsilon}} &= \frac{1 - e^{-\bar{x}} - \bar{x}e^{-\bar{x}}}{\bar{x}} \frac{f(\hat{\varepsilon})}{1 - F(\hat{\varepsilon})} \int_{\hat{\varepsilon}}^{\bar{\varepsilon}} (\varepsilon - \hat{\varepsilon}) f(\varepsilon) d\varepsilon - \eta(\bar{x}) (1 - F(\hat{\varepsilon})) \\ &< \frac{\eta(\bar{x}) f(\hat{\varepsilon})}{1 - F(\hat{\varepsilon})} \Delta(\hat{\varepsilon}), \end{aligned}$$

where

$$\Delta(\hat{\varepsilon}) \equiv \left( \int_{\hat{\varepsilon}}^{\bar{\varepsilon}} (\varepsilon - \hat{\varepsilon})\, f(\varepsilon) d\varepsilon - \frac{(1 - F(\hat{\varepsilon}))^2}{f(\hat{\varepsilon})} \right).$$

Note that $\Delta(\cdot) \to 0$ as $\hat{\varepsilon} \to \bar{\varepsilon}$, and

$$\frac{d\Delta(\hat{\varepsilon})}{d\hat{\varepsilon}} = (1 - F(\hat{\varepsilon})) \left( 1 + \frac{(1 - F(\hat{\varepsilon}))\, f'(\hat{\varepsilon})}{f(\hat{\varepsilon})^2} \right) > 0,$$

by the log concavity of $1 - F(\hat{\varepsilon})$, implying that $\Delta(\hat{\varepsilon}) < 0$ and $\frac{d\Omega(\hat{\varepsilon})}{d\hat{\varepsilon}} < 0$ for all $\hat{\varepsilon} \in (0, \bar{\varepsilon})$. Since $\Omega(\cdot) \to 0$ as $\hat{\varepsilon} \to \bar{\varepsilon}$, $\Omega(\cdot) \to \frac{1-e^{-x}}{x} E(\varepsilon)$ as $\hat{\varepsilon} \to 0$, the monotonicity $\frac{d\Omega(\hat{\varepsilon})}{d\hat{\varepsilon}} < 0$ guarantees that there exists a unique solution $\hat{\varepsilon} \in (0, \bar{\varepsilon})$ to $\Omega(\hat{\varepsilon}) = c$ for any $c \in (0, \eta(x)E(\varepsilon))$. Given this solution, a unique candidate equilibrium price $p > 0$ is pinned down by (11).

Such a candidate equilibrium price is indeed an equilibrium provided that the payoff function in (10) is quasi-concave, which is guaranteed by log-concavity of the demand function. Define

$$\ln D \equiv \ln \left[ 1 - e^{-x(1-F(\hat{\varepsilon}-p+p_i))} \right].$$

We have

$$\frac{\partial \ln D}{\partial p_i} = -\frac{1}{1 - e^{-x(1-F(\hat{\varepsilon}-p+p_i))}} e^{-x(1-F(\hat{\varepsilon}-p+p_i))} x f(\hat{\varepsilon} - p + p_i)$$

and

$$\begin{aligned} \frac{\partial^2 \ln D}{\partial p_i^2} \quad &= -\frac{e^{-x(1-F(\hat{\varepsilon}-p+p_i))} x \left[ x f^2(\hat{\varepsilon} - p + p_i) + f'(\hat{\varepsilon} - p + p_i) \right]}{1 - e^{-x(1-F(\hat{\varepsilon}-p+p_i))}} \\ &\quad -\frac{e^{-2x(1-F(\hat{\varepsilon}-p+p_i))} x^2 f^2(\hat{\varepsilon} - p + p_i)}{(1 - e^{-x(1-F(\hat{\varepsilon}-p+p_i))})^2}. \end{aligned}$$

Inspection of this derivative immediately reveals that $f' \geq 0$ suffices for the existence of equilibrium. Note that the density need not be non-decreasing; what is needed is that the density is not too decreasing. ■

**Comparative statics:** The derivative of price with respect to the reservation value $\hat{\varepsilon}$ is

$$\frac{dp}{d\hat{\varepsilon}} = -\frac{1}{e^{-\bar{x}}} \left[ 1 + \frac{1 - e^{-\bar{x}}}{\bar{x}} \frac{f'(\hat{\varepsilon})(1 - F(\hat{\varepsilon}))}{f(\hat{\varepsilon})^2} \right] < 0,$$

by the log concavity of $1 - F(\hat{\varepsilon})$. Since $\hat{\varepsilon}$ decreases as search cost increases, we obtain the conclusion that price increases in search cost $c$.

The comparative statics of price with respect to $x$ is somewhat more involved because changes in the buyer-seller ratio affects the price directly and indirectly via the reservation value $\hat{\varepsilon}$:

$$\frac{dp}{dx} = \frac{\partial p}{\partial x} + \frac{\partial p}{\partial \hat{\varepsilon}} \frac{\partial \hat{\varepsilon}}{\partial x}.$$

We have

$$\frac{\partial p}{\partial x} = \frac{\bar{x} - 1 + e^{-\bar{x}}}{x^2 e^{-\bar{x}} f(\hat{\varepsilon})} > 0.$$

On the other hand, we know that $\frac{\partial p}{\partial \hat{\varepsilon}} < 0$. Further, it is intuitive (and straightforward to show) that $\frac{\partial \hat{\varepsilon}}{\partial x} < 0$, since a tighter market (a larger $x$) implies a higher chance of being rationed (a lower $\eta$) and a higher effective search cost $\frac{c}{\eta}$ so consumers become less picky and are willing to accept worse products and stop searching. All in all, we conclude that price is increasing in market tightness $x$. The intuition is that firms do not need to compete so hard to retain buyers who visit when market becomes tighter.

**Proposition 1** *Equilibrium price $p$ is increasing in search cost $c$ and in market tightness $x = \frac{B}{S}$.*

**Welfare:** Social welfare is defined per search period and is denoted by $W$. It is given by

$$\begin{aligned} W &= B\left[-c + \eta(\bar{x})(1 - F(\hat{\varepsilon}))[E[\varepsilon - p|\varepsilon \geq \hat{\varepsilon}]]\right] + S\bar{x}\eta(\bar{x})p \\ &= B\left[-c + \eta(\bar{x})(1 - F(\hat{\varepsilon}))E[\varepsilon|\varepsilon \geq \hat{\varepsilon}]\right], \end{aligned} \tag{12}$$

where in the last equality, that the price is canceled out in welfare follows from $S\bar{x} = B(1 - F(\hat{\varepsilon}))$. In (12), observe that a higher reservation value $\hat{\varepsilon}$ may increase the conditional expectation of match value $E[\varepsilon|\varepsilon \geq \hat{\varepsilon}]$, but decreases the probability that a given buyer visit results in a successful match, $\eta(\bar{x})(1 - F(\hat{\varepsilon}))$.

Applying the equilibrium condition (9) to (12), we get

$$W = B\eta(\bar{x}) \int_{\hat{\varepsilon}}^{\bar{\varepsilon}} \hat{\varepsilon} f(\varepsilon) d\varepsilon = S\left(1 - e^{-x[1-F(\hat{\varepsilon})]}\right)\hat{\varepsilon}, \tag{13}$$

where the term $S\left(1 - e^{-x[1-F(\hat{\varepsilon})]}\right)$ represents the total number of matches and $\hat{\varepsilon}$ the reservation match value. The former is decreasing in $\hat{\varepsilon}$ and the latter is increasing in $\hat{\varepsilon}$: a higher reservation value will increase the match value on average but, with more picky buyers, it will decrease the total number of successful matches. In the extreme case as $\hat{\varepsilon} \to \bar{\varepsilon}$, buyers become most picky so that matching becomes impossible and we have $W \to 0$, whereas as $\hat{\varepsilon} \to 0$, the match between product and preference will become most coarse and we have $W \to 0$ (see Figure 1).

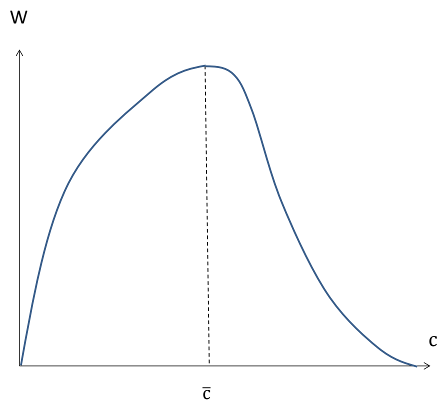


Figure 1: Welfare function

**Proposition 2** *Social welfare $W$ is non monotone in search cost $c$: there exists a unique $\bar{c} \in (0, \eta(x)E(\varepsilon))$ such that $W$ is increasing in $c < \bar{c}$ and $W$ is decreasing in $c > \bar{c}$.*

**Proof.** Since $W \to 0$ as $\hat{\varepsilon} \to 0$, $W \to 0$ as $\hat{\varepsilon} \to \bar{\varepsilon}$ and $W > 0$ for all $\hat{\varepsilon} \in (0, \bar{\varepsilon})$, we must have at least one stationary point, $\frac{dW}{d\hat{\varepsilon}} = 0$. Below we show that such a stationary point has to be unique. Differentiation yields

$$\frac{dW}{d\hat{\varepsilon}} = S\left(1 - e^{-x[1-F(\hat{\varepsilon})]} - xf(\hat{\varepsilon})\hat{\varepsilon}e^{-x[1-F(\hat{\varepsilon})]}\right) \tag{14}$$

$$\frac{d^2W}{d\hat{\varepsilon}^2} = -Sxe^{-x[1-F(\hat{\varepsilon})]}\left[2f(\hat{\varepsilon}) + f'(\hat{\varepsilon})\hat{\varepsilon} + xf(\hat{\varepsilon})^2\hat{\varepsilon}\right]. \tag{15}$$

Now we identify the sign of the terms

$$\Delta_W \equiv 2f(\hat{\varepsilon}) + f'(\hat{\varepsilon})\hat{\varepsilon} + xf(\hat{\varepsilon})^2\hat{\varepsilon}$$

at the stationary point. By the log concavity of $1 - F(\hat{\varepsilon})$,

$$\frac{\Delta_W}{f(\hat{\varepsilon})} > 2 - \frac{f(\hat{\varepsilon})\hat{\varepsilon}}{1 - F(\hat{\varepsilon})} + xf(\hat{\varepsilon})\hat{\varepsilon} = \frac{1}{\bar{x}e^{-\bar{x}}}\left[\bar{x} - 1 + e^{-\bar{x}} + \bar{x}e^{-\bar{x}}\right] > 0,$$

where to derive the equality, we use $\frac{dW}{d\hat{\varepsilon}} = 0$, or $f(\hat{\varepsilon}) = \frac{1-e^{-\bar{x}}}{x\hat{\varepsilon}e^{-\bar{x}}}$. This implies $\frac{d^2W}{d\hat{\varepsilon}^2} < 0$ at $\frac{dW}{d\hat{\varepsilon}} = 0$, which further implies that $\frac{dW}{d\hat{\varepsilon}} < 0$ at $\hat{\varepsilon}$ slightly above the stationary point. Hence, the stationary point should achieve the maximum and not the minimum for the entire domain $\hat{\varepsilon} \in (0, \bar{\bar{\varepsilon}})$, which guarantees that the stationary point is unique. Since $\hat{\varepsilon}$ is monotone decreasing in $c$, we achieve the result. ■

## 4.2 Competitive search

We now consider an alternative setup of competitive search where buyers can observe the prices posted by sellers before they decide which individual seller to visit. Given the limited capacity of sellers, assuming that buyers cannot coordinate their action over which seller to visit implies that we are in the same setup as in standard directed search models. As in the existing literature, we shall focus on symmetric equilibria where sellers play pure-strategies and buyers mixed between sellers (see, e.g. Peters, 1984, 1991, 2001).

In order to derive the equilibrium price, we now consider a deviation by a seller $i$ to a price $p_i$. Now with observable prices, this deviation affects the number of consumers to the seller will receive. Denote by $x_i = x_i(p_i, p)$ the expected queue of buyers to this deviating seller. If a buyer chooses to visit this seller $i$, he expects to get an expected value, denoted by $V(p_i)$, which is given by

$$V(p_i) = -c + \eta(\bar{x}_i)[1 - F(\hat{\varepsilon}_i)][E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_i) - p_i] + [1 - \eta(\bar{x}_i)[1 - F(\hat{\varepsilon}_i)]]\, V(p),$$

where $\bar{x}_i = x_i[1 - F(\hat{\varepsilon}_i)]$ is an effective queue of seller $i$ and $\hat{\varepsilon}_i = p_i + V(p)$ $(= p_i - p + \hat{\varepsilon})$ is the reservation value with respect to price $p_i$. In the above expression, $V(p) = V$ is the equilibrium value of search, as is given by (8), which is expected from visiting a seller posting $p$.

In a directed search equilibrium where buyers are indifferent between all sellers, it must hold that

$$V(p) = V(p_i). \tag{16}$$

This indifference condition can be simplified to

$$\eta(\bar{x})[1 - F(\hat{\varepsilon})][E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}) - p - V(p)] = \eta(\bar{x}_i)[1 - F(\hat{\varepsilon}_i)][E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_i) - p_i - V(p)].$$

Observe that, by (8), the L.H.S. of this equation equals $c$. Let us denote the R.H.S. by $\Psi(x_i, p_i)$. Then we have

$$\begin{aligned}
\Psi(x_i, p_i) &\equiv \eta(\bar{x}_i)[1 - F(\hat{\varepsilon}_i)][E(\varepsilon \mid \varepsilon \geq \hat{\varepsilon}_i) - p_i - V(p)] \\
&= \eta(\bar{x}_i)\left[\int_{\hat{\varepsilon}_i}^{\infty} \varepsilon f(\varepsilon)d\varepsilon - [1 - F(\hat{\varepsilon}_i)]\hat{\varepsilon}_i\right] \\
&= \eta(\bar{x}_i)\int_{\hat{\varepsilon}_i}^{\infty} (\varepsilon - \hat{\varepsilon}_i)f(\varepsilon)d\varepsilon \\
&= \eta(x_i(1 - F(p_i - p + \hat{\varepsilon})))\int_{p_i - p + \hat{\varepsilon}}^{\infty} (\varepsilon - (p_i - p + \hat{\varepsilon}))f(\varepsilon)d\varepsilon.
\end{aligned}$$

Hence, the indifference condition (16) becomes

$$\Psi(x_i, p_i) = c. \tag{17}$$

Solving this equation for $x_i = x_i(p_i, p)$ gives the expected number of buyers who will visit a deviant firm charging price $p_i$.

The deviant's profit function is

$$\Pi_i(p_i, p) = p_i\left[1 - e^{-x_i(p_i,p)[1-F(p_i-p+\hat{\varepsilon})]}\right], \tag{18}$$

where $x_i = x_i(p_i, p)$ is determined by the indifference condition (17). The first order condition is

$$1 - e^{-x_i(1-F(p_i-p+\hat{\varepsilon}))} + p_i e^{-x_i(1-F(p_i-p+\hat{\varepsilon}))}\left[-x_i f(p_i - p + \hat{\varepsilon}) + \frac{\partial x_i}{\partial p_i}[1 - F(p_i - p + \hat{\varepsilon})]\right] = 0.$$

To calculate $\partial x_i/\partial p_i$, we apply the implicit function theorem to equation (17):

$$\frac{\partial x_i}{\partial p_i} = -\frac{\frac{\partial \Psi(x_i,p_i)}{\partial p_i}}{\frac{\partial \Psi(x_i,p_i)}{\partial x_i}},$$

where

$$\begin{aligned}
\frac{\partial \Psi(x_i, p_i)}{\partial p_i} = \; & \frac{1 - e^{-x_i(1-F(\hat{\varepsilon}-p+p_i)} - x_i(1 - F(\hat{\varepsilon} - p + p_i)e^{-x_i(1-F(\hat{\varepsilon}-p+p_i)}}{x_i^2(1 - F(\hat{\varepsilon} - p + p_i))^2} \cdot \\
& x_i f(\hat{\varepsilon} - p + p_i)\int_{p_i-p+\hat{\varepsilon}}^{\infty} (\varepsilon - (p_i - p + \hat{\varepsilon}))f(\varepsilon)d\varepsilon - \eta(x_i(1 - F(p_i - p + \hat{\varepsilon})))\int_{p_i-p+\hat{\varepsilon}}^{\infty} f(\varepsilon)d\varepsilon
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \Psi(x_i, p_i)}{\partial x_i} = \; & -\frac{1 - e^{-x_i(1-F(\hat{\varepsilon}-p+p_i)} - x_i(1 - F(\hat{\varepsilon} - p + p_i)e^{-x_i(1-F(\hat{\varepsilon}-p+p_i)}}{x_i^2(1 - F(\hat{\varepsilon} - p + p_i))^2} \cdot \\
& (1 - F(\hat{\varepsilon} - p + p_i))\int_{p_i-p+\hat{\varepsilon}}^{\infty} (\varepsilon - (p_i - p + \hat{\varepsilon}))f(\varepsilon)d\varepsilon.
\end{aligned}$$

Imposing symmetry, i.e. $p_i = p$ and $x_i = x$, the FOC simplifies to

$$1 - e^{-x(1-F(\hat{\varepsilon}))} + pe^{-x(1-F(\hat{\varepsilon}))} \left[ -xf(\hat{\varepsilon}) + \left.\frac{\partial x_i}{\partial p_i}\right|_{p_i=p} (1 - F(\hat{\varepsilon})) \right] = 0.$$

where

$$\left.\frac{\partial x_i}{\partial p_i}\right|_{p_i=p} = x\frac{f(\hat{\varepsilon})}{1-F(\hat{\varepsilon})} - \frac{(1-e^{-x(1-F(\hat{\varepsilon}))})^2}{[1-e^{-x(1-F(\hat{\varepsilon}))} - x(1-F(\hat{\varepsilon})e^{-x(1-F(\hat{\varepsilon})})]c}.$$

After rearranging we obtain the following candidate equilibrium price:

$$p = \frac{(1-e^{-\bar{x}} - \bar{x}e^{-\bar{x}})xc}{\bar{x}e^{-\bar{x}}(1-e^{-\bar{x}})} \tag{19}$$

**Equilibrium:** A competitive sequential search equilibrium in our economy is described by $(p, \hat{\varepsilon})$ satisfying (9) and (19).

**Theorem 3** *For any $x = \frac{B}{S} \in (0, \infty)$ and $c \in (0, \eta(x)E(\varepsilon))$, there exists a unique solution $(p, \hat{\varepsilon}) > 0$ to (9) and (19).*

**Proof.** The determination of the reservation value $\hat{\varepsilon} > 0$ follows that in the proof of Theorem 2. Proving the existence of equilibrium is extremely tedious because computing the second order derivative of the payoff in (18) involves the computation of the second order derivative of the function $x_i(p_i, p)$, which unfortunately cannot be obtained in closed form. Nevertheless, note that the payoff in (18) is known to be strictly concave in $p_i$ when firms sell homogeneous products (see e.g. Peters, 1984). By continuity, we can pick densities $f$ arbitrarily close to the degenerate density at $\varepsilon = \bar{\varepsilon}$ at the payoff will remain strictly concave. We have also studied the payoff for the case where match values are uniformly distributed and $x = 1$; for such a case, the payoff is strictly concave in a neighbourhood of the equilibrium price. ■

**Comparative statics:** The derivative of price with respect to the reservation value $\hat{\varepsilon}$ is

$$\frac{dp}{d\hat{\varepsilon}} = -\frac{1}{e^{-\bar{x}}} \left[ 1 + \frac{1-e^{-\bar{x}}}{\bar{x}} \frac{f'(\hat{\varepsilon})(1-F(\hat{\varepsilon}))}{f(\hat{\varepsilon})^2} \right] < 0,$$

by the log concavity of $1 - F(\hat{\varepsilon})$. Since $\hat{\varepsilon}$ decreases as search cost increases, we obtain the conclusion that price increases in search cost $c$.

The comparative statics of price with respect to $x$ is somewhat more involved because changes in the buyer-seller ratio affects the price directly and indirectly via the reservation value $\hat{\varepsilon}$:

$$\frac{dp}{dx} = \frac{\partial p}{\partial x} + \frac{\partial p}{\partial \hat{\varepsilon}} \frac{\partial \hat{\varepsilon}}{\partial x}.$$

We have

$$\frac{\partial p}{\partial x} = \frac{\bar{x} - 1 + e^{-\bar{x}}}{x^2 e^{-\bar{x}} f(\hat{\varepsilon})} > 0.$$

On the other hand, we know that $\frac{\partial p}{\partial \hat{\varepsilon}} < 0$. Further, it is intuitive (and straightforward to show) that $\frac{\partial \hat{\varepsilon}}{\partial x} < 0$, since a tighter market (a larger $x$) implies a higher chance of being rationed (a lower $\eta$) and a higher effective search cost $\frac{c}{\eta}$ so consumers become less picky and are willing to accept worse products and stop searching. All in all, we conclude that price is increasing in market tightness $x$. The intuition is that firms do not need to compete so hard to retain buyers who visit when market becomes tighter.

**Corollary 4** *Equilibrium price $p$ is increasing in search cost $c$ and in market tightness $x = \frac{B}{S}$.*

**Welfare:** Social welfare is the same as (12) and so the welfare implication of search costs in a competitive sequential search equilibrium is the same as the one described before

**Proposition 3** *Social welfare $W$ is non monotone in search cost $c$: there exists a unique $\bar{c} \in (0, \eta(x)E(\varepsilon))$ such that $W$ is increasing in $c < \bar{c}$ and $W$ is decreasing in $c > \bar{c}$. With the participation constraint of buyers, (**??**). A search equilibrium exists if and only if $c \leq \bar{c}$. Social welfare $W$ is increasing in all $c < \bar{c}$.*

**Proof.** TO BE COMPLETED.

Since $W \to 0$ as $\hat{\varepsilon} \to 0$, $W \to 0$ as $\hat{\varepsilon} \to \bar{\varepsilon}$ and $W > 0$ for all $\hat{\varepsilon} \in (0, \bar{\varepsilon})$, we must have at least one stationary point, $\frac{dW}{d\hat{\varepsilon}} = 0$. Below we show that such a stationary point has to be unique. Differentiation yields

$$\frac{dW}{d\hat{\varepsilon}} = S\left(1 - e^{-x[1-F(\hat{\varepsilon})]} - xf(\hat{\varepsilon})\hat{\varepsilon}e^{-x[1-F(\hat{\varepsilon})]}\right) \tag{20}$$

$$\frac{d^2W}{d\hat{\varepsilon}^2} = -Sxe^{-x[1-F(\hat{\varepsilon})]}\left[2f(\hat{\varepsilon}) + f'(\hat{\varepsilon})\hat{\varepsilon} + xf(\hat{\varepsilon})^2\hat{\varepsilon}\right]. \tag{21}$$

Now we identify the sign of the terms

$$\Delta_W \equiv 2f(\hat{\varepsilon}) + f'(\hat{\varepsilon})\hat{\varepsilon} + xf(\hat{\varepsilon})^2\hat{\varepsilon}$$

at the stationary point. By the log concavity of $1 - F(\hat{\varepsilon})$,

$$\frac{\Delta_W}{f(\hat{\varepsilon})} > 2 - \frac{f(\hat{\varepsilon})\hat{\varepsilon}}{1 - F(\hat{\varepsilon})} + xf(\hat{\varepsilon})\hat{\varepsilon} = \frac{1}{\bar{x}e^{-\bar{x}}}\left[\bar{x} - 1 + e^{-\bar{x}} + \bar{x}e^{-\bar{x}}\right] > 0,$$

where to derive the equality, we use $\frac{dW}{d\hat{\varepsilon}} = 0$, or $f(\hat{\varepsilon}) = \frac{1-e^{-\bar{x}}}{x\hat{\varepsilon}e^{-\bar{x}}}$. This implies $\frac{d^2W}{d\hat{\varepsilon}^2} < 0$ at $\frac{dW}{d\hat{\varepsilon}} = 0$, which further implies that $\frac{dW}{d\hat{\varepsilon}} < 0$ at $\hat{\varepsilon}$ slightly above the stationary point. Hence, the stationary point should achieve the maximum and not the minimum for the entire domain $\hat{\varepsilon} \in (0, \bar{\hat{\varepsilon}})$, which guarantees that the stationary point is unique. Since $\hat{\varepsilon}$ is monotone decreasing in $c$, we achieve the result. ■

# 5 Free entry equilibrium

In this Section we study the long-run equilibrium and ask whether the number of firms that enter the market is socially optimal. Since products are differentiated in our model, we should interpret this question as whether the market provides an excessive or insufficient amount of variety.

## 5.1 Random search

Let $K$ denote the entry cost. A long-run equilibrium is a triplet $\{x_f, p_f, \hat{\varepsilon}_f\}$ such that the following free-entry equation holds:

$$\pi \equiv p(1 - e^{-x(1-F(\hat{\varepsilon}))}) - K = 0 \tag{22}$$

and $\hat{\varepsilon}_f$ and $p_f$ continue to be given by (9) and (11), respectively.

We now show that for any search cost $c$ a long run equilibrium exists provided that the entry cost $K$ and is low enough. To see this, note that, after using a long-run equilibrium is given by the solution to the following system of equations:

$$\frac{(1 - e^{-x(1-F(\hat{\varepsilon}))})^2}{xf(\hat{\varepsilon})e^{-x(1-F(\hat{\varepsilon}))}} - E = 0 \tag{23}$$

$$\frac{1 - e^{-x(1-F(\hat{\varepsilon}))}}{x(1 - F(\hat{\varepsilon}))}\int_{\hat{\varepsilon}}^{\bar{\varepsilon}} (\varepsilon - \hat{\varepsilon})f(\varepsilon)d\varepsilon - c = 0 \tag{24}$$

where we have used squat Equation (23) implicitly defines a relationship between $x$ and $\hat{\varepsilon}$. Let us denote such a relation as $x = h_1(\hat{\varepsilon})$. Likewise, equation (24) implicitly defines a relationship between $x$ and $\hat{\varepsilon}$ that we denote $x = h_2(\hat{\varepsilon})$.

We now note the following facts about the functions $h_1$ and $h_2$. First, $h_1(0)$ is strictly positive; moreover $h_1$ is increasing in $\hat{\varepsilon}$. Second, $h_2(0)$ is strictly positive and the function$h_2$ decreases in $\hat{\varepsilon}$, with $h_2(\overline{\varepsilon}) = -c < 0$. Therefore, an equilibrium exists provided that $h_1(0) < h_2(0)$.

We now note that $h_1(0)$ is the solution to

$$\frac{(1-e^{-x})^2}{xe^{-x}} = Ef(0), \tag{25}$$

while $h_2(0)$ solves

$$\frac{1-e^{-x}}{x} - \frac{c}{E[\varepsilon]}, \tag{26}$$

where $E[\varepsilon] = \int_0^{\overline{\varepsilon}} \varepsilon f(\varepsilon) d\varepsilon$. Observe further that the L.H.S. of (25) starts at zero and increases in $x$; by contrast the L.H.S. of (26) starts at 1 and decreases in $x$. As a result we conclude that, for any entry cost $c$, a long-run equilibrium exists provided that the entry cost $K$ is sufficiently low. In particular, let us denote the solution to in $x$ to equation (26) by $\hat{x}$. Then, a free entry equilibrium exists provided that

$$K < \overline{K} \equiv \frac{1}{f(0)} \frac{c}{E[\varepsilon]} \frac{1-e^{-\hat{x}}}{e^{-\hat{x}}}.$$

An increase in search costs shifts the $h_2$ function downwards; as a result, higher search costs lead to an increase in the number of active firms and to decrease in consumer reservation value. With free entry the negative effect of an increase in search cost is softened by entry of firms. An increase in the entry cost shifts the $h_1$ function to the left, which implies that there is exit of firms and a corresponding fall in the reservation value.

**Proposition 4** *For any search cost $c < \overline{c}$ and entry cost $K < \overline{K}$ a long-run equilibrium exists. The long-run equilibrium number of sellers increases in search cost $c$ and decreases in entry cost $K$.*

We now examine whether the market provides the right incentives to the firms to enter. In dealing with this question we assume that the social planner faces the same technology of search and production as the market players; likewise, the social planner faces the same coordination frictions. From the expression in (13), under free entry the welfare formula is:

$$W = \frac{B}{x}\left[\hat{\varepsilon}\left(1 - e^{-x(1-F(\hat{\varepsilon}))}\right) - K\right] \tag{27}$$

Taking the FOC with respect to $x$ gives:

$$\begin{aligned} \frac{dW}{dx} = \frac{1}{x}\left[\hat{\varepsilon}e^{-x(1-F(\hat{\varepsilon}))}(1-F(\hat{\varepsilon})) + \frac{d\hat{\varepsilon}}{dx}\left(1 - e^{-x(1-F(\hat{\varepsilon}))} - x\hat{\varepsilon}f(\hat{\varepsilon})e^{-x(1-F(\hat{\varepsilon}))}\right)\right] \\ -\frac{1}{x^2}\left[\hat{\varepsilon}\left(1 - e^{-x(1-F(\hat{\varepsilon}))}\right) - K\right] = 0, \end{aligned} \tag{28}$$

where $d\hat{\varepsilon}/dx$ can be computed from the consumer equilibrium condition.

We should note that when the search cost is sufficiently high, then the free entry equilibrium number of firms is excessive from the point of view of welfare maximization (see Figure 2). To see this, recall that when $c \to \bar{c}$, then the equilibrium price $p \to \hat{\varepsilon}$. Using this, the zero profits condition gives $K = \hat{\varepsilon}(1 - e^{-x(1-F(\hat{\varepsilon}))})$. Moreover, because the equilibrium price satisfies the FOC (), in the limit when $p \to \hat{\varepsilon}$ we therefore have $1 - e^{-x(1-F(\hat{\varepsilon}))} - x\hat{\varepsilon}f(\hat{\varepsilon})e^{-x(1-F(\hat{\varepsilon}))} = 0$. Then, denoting the free entry level of $x$ by $x_f$ we can write that

$$\lim_{c\to\bar{c}} \left.\frac{dW}{dx}\right|_{x=x_f} = \frac{1}{x_f}\hat{\varepsilon}e^{-x_f(1-F(\hat{\varepsilon}))}(1-F(\hat{\varepsilon})) > 0.$$

From this we conclude that at the free entry long-run equilibrium, the welfare function is increasing in $x$ so the social planner prefers less entry.

**Proposition 5** *For any cost of entry $K$, there exists a sufficiently high search cost such that entry in long-run equilibrium is excessive from the point of view of welfare.*

The analysis of entry when the search cost is low, has proven to be more difficult. Our analysis nevertheless suggests instead insufficient entry. To study this question, we first compute the following derivative:

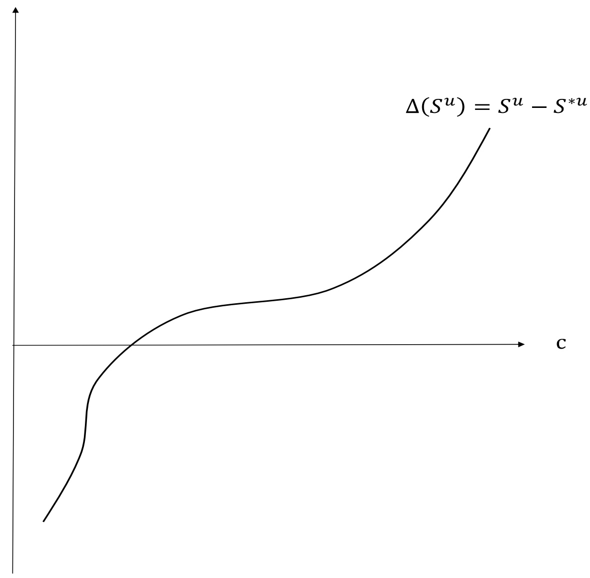


Figure 2: Excessive entry in random search equilibrium

$$\frac{d\hat{\varepsilon}}{dx} = -\frac{\frac{d\eta}{dx}\int_{\hat{\varepsilon}}^{\overline{\varepsilon}}(\varepsilon-\hat{\varepsilon})f(\varepsilon)d\varepsilon}{\frac{d\eta}{d\hat{\varepsilon}}\int_{\hat{\varepsilon}}^{\overline{\varepsilon}}(\varepsilon-\hat{\varepsilon})f(\varepsilon)d\varepsilon - \eta(1-F(\hat{\varepsilon})}$$
$$= \frac{\frac{d\eta}{dx}}{\frac{d\eta}{d\hat{\varepsilon}} - \frac{\eta(1-F(\hat{\varepsilon}))}{\int_{\hat{\varepsilon}}^{\overline{\varepsilon}}(\varepsilon-\hat{\varepsilon})f(\varepsilon)d\varepsilon}} \tag{29}$$

where

$$\frac{d\eta}{dx} = -\frac{1-e^{-x(1-F(\hat{\varepsilon}))}+x(1-F(\hat{\varepsilon}))e^{-x(1-F(\hat{\varepsilon}))}}{x^2(1-F(\hat{\varepsilon}))} \tag{30}$$
$$\frac{d\eta}{d\hat{\varepsilon}} = \frac{1-e^{-x(1-F(\hat{\varepsilon}))}+x(1-F(\hat{\varepsilon}))e^{-x(1-F(\hat{\varepsilon}))}}{x(1-F(\hat{\varepsilon}))^2}f(\hat{\varepsilon}). \tag{31}$$

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Now note that when $c \to 0$, $\hat{\varepsilon} \to \overline{\varepsilon}$ hence $F(\hat{\varepsilon}) \to 1$. This implies that for the numerator of (29) we have

$$\lim_{c\to 0}\frac{d\eta}{dx} = -\frac{2}{x}$$

and for the denominator we can write the following:

$$\lim_{c\to 0}\left[\frac{d\eta}{d\hat{\varepsilon}}\right] = f(\overline{\varepsilon}) \times \lim_{c\to 0}\frac{1-e^{-x(1-F(\hat{\varepsilon}))}+x(1-F(\hat{\varepsilon}))e^{-x(1-F(\hat{\varepsilon}))}}{x(1-F(\hat{\varepsilon}))^2}$$

while

$$\lim_{c\to 0}\left[\frac{\eta(1-F(\hat{\varepsilon}))}{\int_{\hat{\varepsilon}}^{\overline{\varepsilon}}(\varepsilon-\hat{\varepsilon})f(\varepsilon)d\varepsilon}\right]=f(\overline{\varepsilon})\times\lim_{c\to 0}\left[\frac{e^{-x(1-F(\hat{\varepsilon}))}}{1-F(\hat{\varepsilon}))}\right]$$

so putting things together we get

$$\lim_{c\to 0}\left[\frac{d\eta}{d\hat{\varepsilon}}-\frac{\eta(1-F(\hat{\varepsilon}))}{\int_{\hat{\varepsilon}}^{\overline{\varepsilon}}(\varepsilon-\hat{\varepsilon})f(\varepsilon)d\varepsilon}\right]=f(\overline{\varepsilon})\times\lim_{c\to 0}\frac{1-e^{-x(1-F(\hat{\varepsilon}))}}{x(1-F(\hat{\varepsilon}))^2}=\infty.$$

The conclusion is then that

$$\lim_{c\to 0}\frac{d\hat{\varepsilon}}{dx}=0.$$

Now are ready to prove the result that when search cost is sufficiently small the free entry equilibrium number of firms is insufficient. For this we note that:

$$\lim_{c\to 0}\left.\frac{dW}{dx}\right|_{x=x_f}=-\frac{1}{x^2}\left[(\hat{\varepsilon}-p)\left(1-e^{-x(1-F(\hat{\varepsilon}))}\right)\right]=0$$

Comparing the social planner solution with the free entry condition is complicated for general distributions of match values. We will assume from now on that $F$ is the uniform distribution on $[0,1]$. In that case, after simplification, the socially optimal equilibrium is given by the solution to the system:

$$-(1-K)e^{2(1-\hat{\varepsilon})x}+e^{x(1-\hat{\varepsilon})}\left[(1+K)(1-\hat{\varepsilon})x+2-K\right]-(1-\hat{\varepsilon})x-1=0 \tag{32}$$

$$\frac{1-e^{-x(1-F(\hat{\varepsilon}))}}{x(1-F(\hat{\varepsilon}))}\int_{\hat{\varepsilon}}^{1}(\varepsilon-\hat{\varepsilon})f(\varepsilon)-c=0 \tag{33}$$

Numerically, we have observed that for low search costs, the free entry number of firms is insufficient. For example if we set $K=0.1$ and the search cost equal to $c=0.001$, we obtain a free entry equilibrium $x_f=22.94$ and $\hat{\varepsilon}_f=0.949$ while the social optimum is $x_o=13.81$ and $\hat{\varepsilon}_o=0.946$, with associated welfare losses of around 19%.

## 5.2 Competitive search

**Proposition 6** *In a competitive sequential search equilibrium, the long-run equilibrium number of sellers is insufficient from the point of view of welfare maximisation provided that when $c<\overline{c}$; in the limit when $c\to\overline{c}$ entry is efficient.*

**Proof.** I will do the proof of the second part.

As shown above, welfare is:

$$W = \frac{B}{x}\left[\hat{\varepsilon}\left(1 - e^{-x(1-F(\hat{\varepsilon}))}\right) - K\right] \tag{34}$$

and the FOC with respect to $x$ gives:

$$\frac{dW}{dx} = \frac{1}{x}\left[\hat{\varepsilon}e^{-x(1-F(\hat{\varepsilon}))}(1-F(\hat{\varepsilon})) + \frac{d\hat{\varepsilon}}{dx}\left(1 - e^{-x(1-F(\hat{\varepsilon}))} - x\hat{\varepsilon}f(\hat{\varepsilon})e^{-x(1-F(\hat{\varepsilon}))}\right)\right] - \frac{1}{x^2}\left[\hat{\varepsilon}\left(1 - e^{-x(1-F(\hat{\varepsilon}))}\right) - K\right] = 0, \tag{35}$$

where $d\hat{\varepsilon}/dx$ can be computed from the consumer equilibrium condition.

Under ex-ante competition, the price is given by

$$p = \frac{1 - e^{-x(1-F(\hat{\varepsilon}))}}{e^{-x(1-F(\hat{\varepsilon}))}} \frac{1}{xf(\hat{\varepsilon}) - \left.\frac{\partial x_i}{\partial p_i}\right|_{p_i=p}(1-F(\hat{\varepsilon}))}$$

where

$$\left.\frac{\partial x_i}{\partial p_i}\right|_{p_i=p} = \frac{\frac{1-e^{-x(1-F(\hat{\varepsilon}))}-x(1-F(\hat{\varepsilon}))e^{-x(1-F(\hat{\varepsilon}))}}{x^2(1-F(\hat{\varepsilon}))^2}x\frac{f(\hat{\varepsilon})}{1-F(\hat{\varepsilon})}c - \eta^2}{\frac{1-e^{-x(1-F(\hat{\varepsilon}))}-x(1-F(\hat{\varepsilon}))e^{-x(1-F(\hat{\varepsilon}))}}{x^2(1-F(\hat{\varepsilon}))^2}c}.$$

When $c \to \overline{c}$, then the equilibrium price $p \to \hat{\varepsilon}$. Using this, the zero profits condition gives $K = \hat{\varepsilon}(1 - e^{-x(1-F(\hat{\varepsilon}))})$ so the last term of (35) cancels out.

From the price equation we obtain that when $p \to \hat{\varepsilon}$ :

$$1 - e^{-x(1-F(\hat{\varepsilon}))} - x\hat{\varepsilon}f(\hat{\varepsilon})e^{-x(1-F(\hat{\varepsilon}))} = \hat{\varepsilon}e^{-x(1-F(\hat{\varepsilon}))}(1-F(\hat{\varepsilon}))\left.\frac{\partial x_i}{\partial p_i}\right|_{p_i=p}.$$

Then, using this relation in (35), we can write that

$$\lim_{c\to\overline{c}}\left.\frac{dW}{dx}\right|_{x=x_f} = \frac{1}{x_f}\hat{\varepsilon}e^{-x_f(1-F(\hat{\varepsilon}))}(1-F(\hat{\varepsilon}))\left[1 - \left.\frac{d\hat{\varepsilon}}{dx}\right|_{x=x_f}\left.\frac{\partial x_i}{\partial p_i}\right|_{x=x_f,p_i=p}\right],$$

where $x_f$ denotes the free entry number of sellers.

It is a matter of a few calculations to show that $\left.\frac{d\hat{\varepsilon}}{dx}\right|_{x=x_f}\left.\frac{\partial x_i}{\partial p_i}\right|_{x=x_f,p_i=p} = 1$. In fact, from (29) we have

$$\left.\frac{d\hat{\varepsilon}}{dx}\right|_{x=x_f} = -\frac{\frac{d\eta}{dx}c}{\frac{d\eta}{d\hat{\varepsilon}}c - \eta(1-F(\hat{\varepsilon})}$$

and we can write

$$\left.\frac{\partial x_i}{\partial p_i}\right|_{x=x_f,p_i=p} = \frac{\frac{d\eta}{d\hat{\varepsilon}}\frac{c}{1-F(\hat{\varepsilon})} - \eta}{-\frac{d\eta}{dx}\frac{c}{1-F(\hat{\varepsilon})}}$$

and the result follows. ■

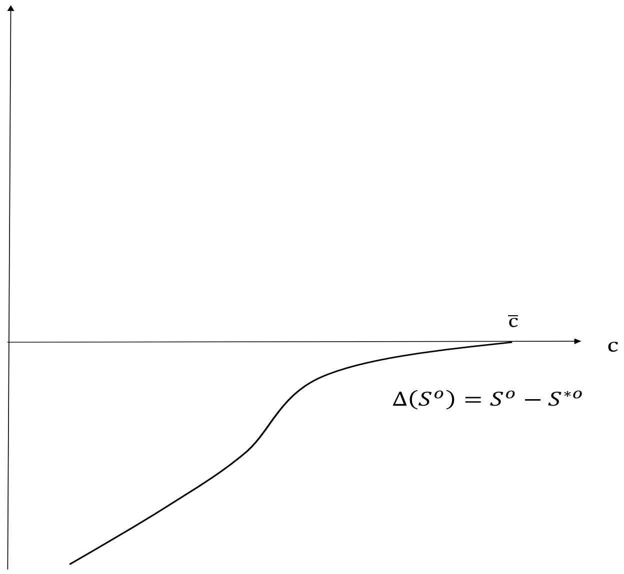


Figure 3: Insufficient entry in competitive sequential search equilibrium

The dependence of the number of sellers in a competitive sequential search equilibrium is depicted in Figure 3.

Comparing the last two propositions, we obtain the following results:

**Proposition 7** *There exists a subset of search costs in which seller entry attains higher welfare with random search than with competitive search.*

The comparison of the two equilibria in terms of the number of sellers and welfare is illustrated in Figure 4.

## 6 Conclusions

We study a competitive differentiated products consumer search market with capacity constrained sellers. We prove that a symmetric equilibrium exists and is unique. The higher the number of

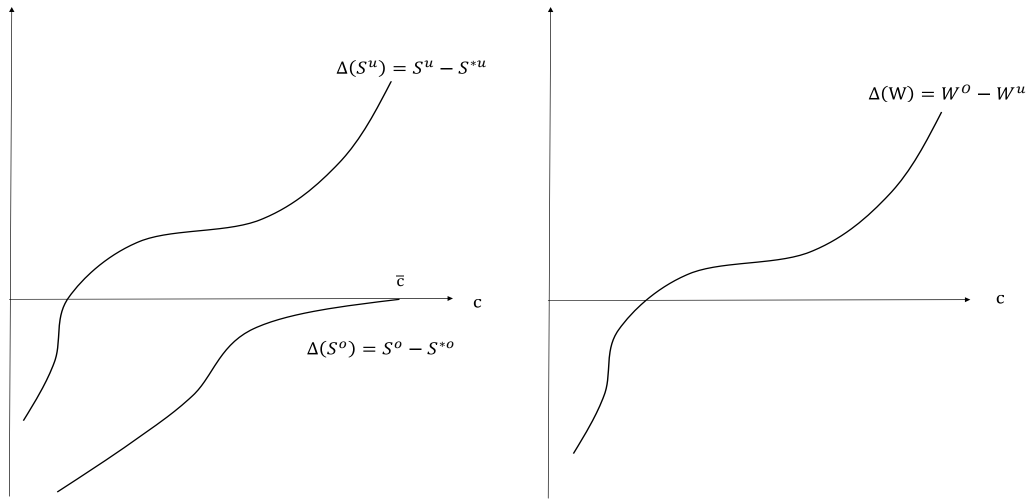


Figure 4: Comparison

buyers relative to the sellers, the higher the equilibrium price. Higher search costs also result in higher prices. Welfare, however, is non-monotonic in search costs. The long-run equilibrium number of sellers increases in search cost and decreases in entry cost.

Our paper, by allowing for differentiated products, is the first to deal with product differentiation in the competitive search equilibrium literature. Product differentiation has the property that we can study the role of capacity constraints under the standard search paradigm, i.e. where consumers do not observe deviation prices without search and to compare it with the case of ex-ante competition.

Under the standard search paradigm, the free entry number of sellers is insufficient while for large search cost it is excessive. The next step is to check whether ex-ante competition can align the private and the social incentives when products are differentiated. For this, the equilibrium price must induce the right amount of sequential search by buyers.

## Appendix: Derivation of Offer Probability $\eta$

Note that the number of buyers visiting a firm $n$ follows a Poisson distribution, Prob.$(n = i) = \frac{x^i e^{-x}}{i!}$. Given that all buyers are using a reservation strategy with $\hat{\varepsilon}$, we have

$$\begin{aligned}
\eta &= e^{-x} + xe^{-x}\left[(1-F(\hat{\varepsilon}))\frac{1}{2} + F(\hat{\varepsilon})\right] + \frac{x^2e^{-x}}{2}\left[(1-F(\hat{\varepsilon}))^2\frac{1}{3} + 2F(\hat{\varepsilon})(1-F(\hat{\varepsilon}))\frac{1}{2} + F(\hat{\varepsilon})^2\right] + .... \\
&\quad + \frac{x^ie^{-x}}{i!}[(1-F(\hat{\varepsilon}))^i\frac{1}{i+1} + iF(\hat{\varepsilon})(1-F(\hat{\varepsilon}))^{i-1}\frac{1}{i} + \frac{i!}{2!(i-2)!}F(\hat{\varepsilon})^2(1-F(\hat{\varepsilon}))^{i-2}\frac{1}{i-1} + .... \\
&\quad + \frac{i!}{j!(i-j)!}F(\hat{\varepsilon})^j(1-F(\hat{\varepsilon}))^{i-j}\frac{1}{i-j+1} + .... + F(\hat{\varepsilon})^i] + .....
\end{aligned}$$

To see how it works, consider the case $i = 2$ where two other buyers show up at the firm, which occurs with probability $\frac{x^2e^{-x}}{2}$ (the their term in the above expression). If both of these buyers appear ro like the product offered by this firm, which happens with probability $(1 - F(\hat{\varepsilon}))^2$, then the given buyer will receive an offer with probability $\frac{1}{3}$. If one of them likes it but the other of them does not like it, which occurs in 2 ways and with probability $F(\hat{\varepsilon})(1 - F(\hat{\varepsilon}))$, then the given buyer will be offered with probability $\frac{1}{2}$. If none of the other buyers happen to like it, which happens with probability $F(\hat{\varepsilon})^2$, then the given buyer will be offered with probability one. By induction, the same logic applies to general case $i$ other buyers show up (with probability $\frac{x^ie^{-x}}{i!}$): if $j \leq i$ of the other buyers turn out to like the firm's product, which comes in $\frac{i!}{j!(i-j)!}$ ways and occurs with probability $F(\hat{\varepsilon})^j(1 - F(\hat{\varepsilon}))^{i-j}$, then the given buyer will be offered with probability $\frac{1}{i-j+1}$.

Note that we can simplify the terms,

$$\begin{aligned}
& (1-F(\hat{\varepsilon}))^i\frac{1}{i+1} + iF(\hat{\varepsilon})(1-F(\hat{\varepsilon}))^{i-1}\frac{1}{i} + \frac{i!}{2!(i-2)!}F(\hat{\varepsilon})^2(1-F(\hat{\varepsilon}))^{i-2}\frac{1}{i-1} \\
& \quad + \frac{i!}{j!(i-j)!}F(\hat{\varepsilon})^j(1-F(\hat{\varepsilon}))^{i-j}\frac{1}{i-j+1} + .... + F(\hat{\varepsilon})^i \\
= {} & \frac{1}{(i+1)(1-F(\hat{\varepsilon}))}\sum_{j=0}^{i}\frac{(i+1)!}{j!(i+1-j)!}F(\hat{\varepsilon})^j(1-F(\hat{\varepsilon}))^{i+1-j} \\
= {} & \frac{1}{(i+1)(1-F(\hat{\varepsilon}))}\left[\sum_{j=0}^{i+1}\frac{(i+1)!}{j!(i+1-j)!}F(\hat{\varepsilon})^j(1-F(\hat{\varepsilon}))^{i+1-j} - F(\hat{\varepsilon})^{i+1}\right] \\
= {} & \frac{1-F(\hat{\varepsilon}))^{i+1}}{(i+1)(1-F(\hat{\varepsilon}))}.
\end{aligned}$$

Using this simplification, we have

$$\eta = \sum_{i=0}^{\infty} \frac{x^i e^{-x}}{i!} \frac{1 - F(\hat{\varepsilon})^{i+1}}{(i+1)(1 - F(\hat{\varepsilon}))} = \frac{1}{x(1 - F(\hat{\varepsilon}))} \sum_{i=0}^{\infty} \frac{x^{i+1} e^{-x} (1 - F(\hat{\varepsilon})^{i+1})}{(i+1)!}.$$

Setting $h \equiv i + 1$, it is further simplified to

$$\begin{aligned}
\eta &= \frac{1}{x(1 - F(\hat{\varepsilon}))} \sum_{h=0}^{\infty} \left[ \frac{x^h e^{-x}}{h!} - \frac{[xF(\hat{\varepsilon})]^h e^{-x}}{h!} \right] = \frac{1}{x(1 - F(\hat{\varepsilon}))} \left[ 1 - e^{-x(1-F(\hat{\varepsilon}))} \sum_{h=0}^{\infty} \frac{[xF(\hat{\varepsilon})]^h e^{-xF(\hat{\varepsilon})}}{h!} \right] \\
&= \frac{1 - e^{-x(1-F(\hat{\varepsilon}))}}{x(1 - F(\hat{\varepsilon}))}.
\end{aligned}$$

■

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